$\mathbb{F}_{p^2}$-maximal curves with many automorphisms are Galois-covered by the Hermitian curve

Daniele Bartoli

University of Perugia

(Joint work with Maria Montanucci and Fernando Torres)

The Hermitian curve $\mathcal{H}_q : y^q + y = x^{q+1}$, for a prime power q, is the best known example of $\mathbb{F}_{q^2}$-maximal curve, i.e curve whose number $N(\mathcal{H}_q)$ of $\mathbb{F}_{q^2}$-rational points attains the Hasse-Weil upper bound $N(\mathcal{H}_q) = q^2 + 1 + 2g(\mathcal{H}_q)q$. Each curve $\mathcal{Y}$ which is covered by an $\mathbb{F}_{q^2}$-maximal curve is also $\mathbb{F}_{q^2}$-maximal.

It is an open problem to decide whether any $\mathbb{F}_{p^2}$-maximal curve is $\mathbb{F}_{p^2}$-covered by the Hermitian curve $\mathcal{H}_p$ or not. We give an affirmative answer for $\mathbb{F}_{p^2}$-maximal curves $\mathcal{X}$ having a large automorphism group, showing that if

$$|Aut(\mathcal{X})| > 84(g(\mathcal{X}) - 1)$$

then $\mathcal{X}$ is Galois covered by $\mathcal{H}_p$.

Also, we show that this result does not extend to curves whose full automorphism group satisfies $|Aut(\mathcal{X})| \leq 84(g(\mathcal{X}) - 1)$, as we construct an $\mathbb{F}_{71^2}$-maximal curve $\mathcal{F}$ of genus 7, having a Hurwitz automorphism group of order 504 which is not Galois covered by $\mathcal{H}_{71}$. The curve $\mathcal{H}$ is the positive characteristic analog of the so called Fricke-MacBeath curve in zero characteristic, and it is the first known example of $\mathbb{F}_{p^2}$-maximal curve which is not Galois-covered by the Hermitian curve $\mathcal{H}_p$.