Binary self-dual codes with a prescribed finite imprimitive permutation group

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(This relates to joint work with Wolfgang Knapp, Mathematisches Institut,
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Abstract

Given a representation of group elements of a group G by permutations one can work modulo p, p a prime, and obtain a representation of G on a vector space V over $GF(p)$. The invariant submodules (i.e., the subspaces of V taken into themselves by every group element) are then all the p-ary codes C for which G is a subgroup of the automorphism group of C.

One of the questions of current interest in coding theory is the following: given a finite non-solvable permutation group G acting transitively on a set Ω, under what conditions on G are self-dual codes invariant under G existent or nonexistent? In the talk, this problem is investigated under the hypothesis that the group G is a finite imprimitive permutation group.

This talk will give an introduction to this fascinating interplay by focusing on examples, chosen mostly for the illustration of a tool to construct linear code using representation theoretical arguments. In particular, we will determine all binary codes of length 552 which admit the sporadic simple group Co_3 as an imprimitive transitive permutation group. Our aim will be to illustrate the results by using representation theoretical arguments and to discuss the combinatorial properties of the codes as well as their relation to some special properties of the Leech lattice group Co_3. For all codes (with two exceptions) we obtain the weight enumerators and in many interesting cases the classification of codewords under the action of the group Co_3. The two exceptions refer to binary self-dual codes, each of which with minimum weight 12.

References

[1] W. Knapp and B. G. Rodrigues. On the binary codes of length 552 which admit the simple group Co_3 as a transitive permutation group. Commun. in Algebra. **51** (4) 2023, 1451 - 1461.