

q -Analogues of graphs in polar spaces

Brecht Verbeken

Brecht.Verbeken@UGent.be

Department of Mathematics: Analysis, Logic and Discrete Mathematics, Ghent University,
Belgium

Abstract. In this talk, q -analogues of graphs will be defined and studied in finite classical polar spaces. In previous articles, q -analogues were defined of strongly regular, Deza and divisible design graphs in vector spaces [1, 2, 3].

Definition. A q -ary graph $\mathcal{E}$ is a pair (V, E) with $V = [W_1]$ and $E \subseteq [W_2]$, where W is a vector space over $\mathbb{F}_q$ and V and E are called the vertex set and edge set of $\mathcal{E}$, respectively. Two distinct vertices $X, Y \in V$ are said to be adjacent whenever the plane $\langle X, Y \rangle$ belongs to E . A q -ary graph $\mathcal{E}$ is k -regular if, for every vertex X , the neighbourhood $N_{\mathcal{E}}(X)$ forms a $(k + 1)$ -dimensional subspace.

In [1], an example related to the symplectic polar space was given. As there is a natural embedding of the vertices and edges in the projective space, this yields the question if q -analogues are also possible in the polar setting. In this setting, the following definition holds.

Definition. Let $\mathcal{P}$ be a polar space embedded in the projective space associated with a vector space W over $\mathbb{F}_q$. A q -ary graph over $\mathcal{P}$ is a pair (V, E) , where $V \subseteq [W_1]$ and $E \subseteq [W_2]$, such that the projective points corresponding to V are precisely the projective points of $\mathcal{P}$ and the elements of E correspond to a subset of the projective lines of $\mathcal{P}$. Two distinct vertices $X, Y \in V$ are said to be adjacent if the projective line $\langle X, Y \rangle$ belongs to E . Furthermore, a q -ary graph is called regular if the neighbourhood of every vertex is a subspace of the same dimension that is completely contained in $\mathcal{P}$.

In this talk, we will study the q -analogues of strongly regular, Deza and divisible design graphs in polar spaces. The close link with spreads on polar spaces will be presented as well as an example of a q -analogue on the parabolic quadric $Q(6, 2)$ that follows from the split Cayley hexagon $\mathcal{H}(2)$.

Keywords: Finite Geometry, q -analogues, Polar spaces, graphs.

References

- [1] M. Braun, D. Crnković, M. De Boeck, V. Mikulić Crnković and A. Švob, q -analogues of strongly regular graphs. *Linear Algebra and its Applications* 693 (2024), 362–373.
- [2] D. Crnković, M. De Boeck, F. Pavese and A. Švob, q -analogues of divisible design graphs and Deza graphs. *Journal of Combinatorial Theory, Series A* 215 (2025), Paper No. 106047, 18 pp.
- [3] D. Crnković, V. Mikulić Crnković, A. Švob and M. Zubović Žutolija, Transitive regular q -analogues of graphs. *The Art of Discrete and Applied Mathematics* 8 (2025), 1, Paper No. 1.05, 7 pp.