Samoortogonalni kodovi konstruirani pomoću orbitnih matrica blok dizajna

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- **D. Crnković, S. Rukavina**: Construction of block designs admitting an abelian automorphism group

Neka je $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ $2-(v,k,\lambda)$ dizajn i $G \leq Aut(\mathcal{D})$.

Označimo:

$\mathcal{P}_1, \ldots, \mathcal{P}_n$ G-orbite točaka

$\mathcal{B}_1, \ldots, \mathcal{B}_m$ G-orbite blokova

$|\mathcal{P}_r| = \omega_r$, $1 \leq r \leq n$

$|\mathcal{B}_i| = \Omega_i$, $1 \leq i \leq m$

Za $x \in \mathcal{B}$ i $P \in \mathcal{P}$ označimo

$$\langle x \rangle = \{Q \in \mathcal{P} | (Q, x) \in \mathcal{I}\}$$

$$\langle P \rangle = \{y \in \mathcal{B} | (P, y) \in \mathcal{I}\}$$

Za $x \in \mathcal{B}_i$ i $P \in \mathcal{P}_r$ označimo

$$\gamma_{ir} = |\langle x \rangle \cap \mathcal{P}_r|$$

$$\Gamma_{ir} = |\langle P \rangle \cap \mathcal{B}_i|$$

Lema 1.

Neka je $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ blok dizajn, $G \leq Aut(\mathcal{D})$, i neka su $\Omega_i, \omega_r, \Gamma_{ir}, \gamma_{ir}$ definirani kao ranije.
Vrijede sljedeće jednakosti:

$$(a)\ \Omega_i \gamma_{ir} = \omega_r \Gamma_{ir}$$

$$(b)\ \sum_{i=1}^{m} \Gamma_{ir} \gamma_{is} = \lambda \omega_s + \delta_{rs} \cdot (r' - \lambda)$$

gdje je δ_{rs} Kroneckerov simbol.

Propozicija 1.

Neka je $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$ blok dizajn, $G \leq Aut(\mathcal{D})$, i neka su $\Omega_i, \omega_r, \Gamma_{ir}, \gamma_{ir}$ definirani kao ranije.
Vrijede sljedeće jednakosti:

$$\sum_{r=1}^{n} \gamma_{ir} = k \tag{1}$$

$$\sum_{i=1}^{m} \frac{\Omega_i}{\omega_r} \gamma_{ir}\gamma_{is} = \lambda\omega_s + \delta_{rs} \cdot (r' - \lambda)\,. \tag{2}$$

Orbitna matrica

Matrica dimenzija $(m \times n)$ s elementima γ_{ir} koji zadovoljavaju svojstva (1) i (2) se naziva orbitna matrica za parametre (v, k, λ) i orbitnu distribuciju $(\omega_1, \ldots, \omega_n)$, $(\Omega_1, \ldots, \Omega_m)$.

Primjer. 2-(10,4,2) dizajn

Točke:

$$\mathcal{P} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

Blokovi:

1. $\{1, 2, 3, 5\}$
2. $\{1, 2, 5, 6\}$
3. $\{1, 3, 7, 8\}$
4. $\{1, 4, 9, 10\}$
5. $\{1, 5, 7, 9\}$
6. $\{1, 6, 8, 10\}$
7. $\{2, 3, 9, 10\}$
8. $\{2, 4, 7, 8\}$
9. $\{2, 5, 8, 10\}$
10. $\{2, , 6, 7, 9\}$
11. $\{3, 4, 5, 6\}$
12. $\{3, 5, 8, 9\}$
13. $\{3, 6, 7, 10\}$
14. $\{4, 5, 7, 10\}$
15. $\{4, 6, 8, 9\}$

Automorfizam reda 2:

$$\Phi = (1, 2)(3, 4)(5, 6)$$

Primjer-nastavak

$$\Phi = (1,2)(3,4)(5,6)$$

Orbite točaka pod djelovanjem grupe $\langle\Phi\rangle$:

$$\{7\},\quad \{8\},\quad \{9\},\quad \{10\},$$
$$\{1,2\},$$
$$\{3,4\},$$
$$\{5,6\}$$

Orbite blokova pod djelovanjem grupe $\langle\Phi\rangle$:

$$\{1\},\quad \{2\},\quad \{11\},$$
$$\{3,8\},$$
$$\{4,7\},$$
$$\{5,10\},$$
$$\{6,9\},$$
$$\{12,15\},$$
$$\{13,14\}$$

Primjer-nastavak

0	0	0	0	1	1	1	1	0	0
0	0	0	0	1	1	0	0	1	1
0	0	0	0	0	0	1	1	1	1
1	1	0	0	1	0	1	0	0	0
1	1	0	0	0	1	0	1	0	0
0	0	1	1	1	0	0	1	0	0
0	0	1	1	0	1	1	0	0	0
1	0	1	0	1	0	0	0	1	0
1	0	1	0	0	1	0	0	0	1
0	1	0	1	1	0	0	0	0	1
0	1	0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	0	1	0
0	1	1	0	0	0	0	1	0	1
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	0	0	1	1	0

Primjer-nastavak

$$\left[\begin{array}{cccc|ccc}
0 & 0 & 0 & 0 & 2 & 2 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 2 & 2 \\
\hline
1 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 1 & 1
\end{array}\right]$$

Tvrdnja

Neka je $\mathcal{D}$ $2-(v,k,\lambda)$ dizajn sa automorfizmom Φ reda p, gdje je p prim broj. Ako p dijeli $r-\lambda$, tada stupci nefiksnog dijela orbitne matrice dizajna $\mathcal{D}$ za automorfizam Φ generiraju samoortogonalni kod nad F_p.